Time-varying signals: cross- and auto-correlation, correlograms

NEU 466M

Spring 2020

Cross-correlation function for finite-length signals

$$\{g_1, \cdots, g_N\}$$
$$\{h_1, \cdots, h_N\}$$

g, h: time-series of length N

$$C_{g,h}(n) = \sum_{m=1}^{N-n} g^*(m)h(m+n)$$

average over *(N-|n|)* terms

Total length of cross-correlation: *2N-1*
Zero-shifted entry: *N*

Properties of the cross-correlation

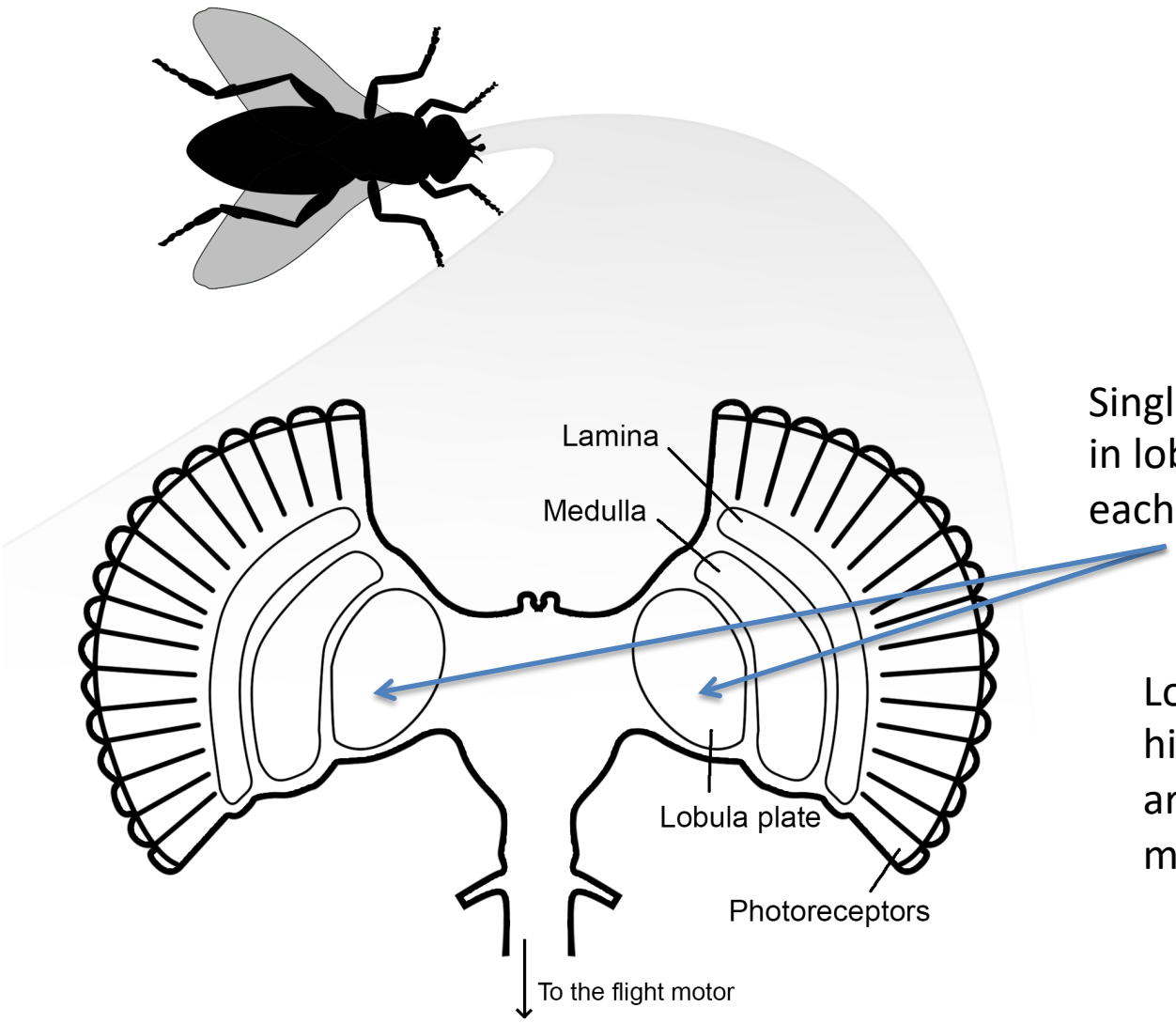
- $C_{gh}(n) \neq C_{hg}(n)$: does not commute (contrast with covariance).
- In fact, $C_{gh}(n) = C_{hg}(-n)$: shifting *h* to *right* relative to *g:* equivalent to shifting *g* to *left* relative to *h*. (Same plot, flipped time axis.)
- Ordering matters: tells which signal leads the other.

Application

SPIKE TRAINS: CROSS-CORRELATION AND SIMPLE STATISTICS

Motion detection in the blowfly

H1 neuron: horizontal motion sensing

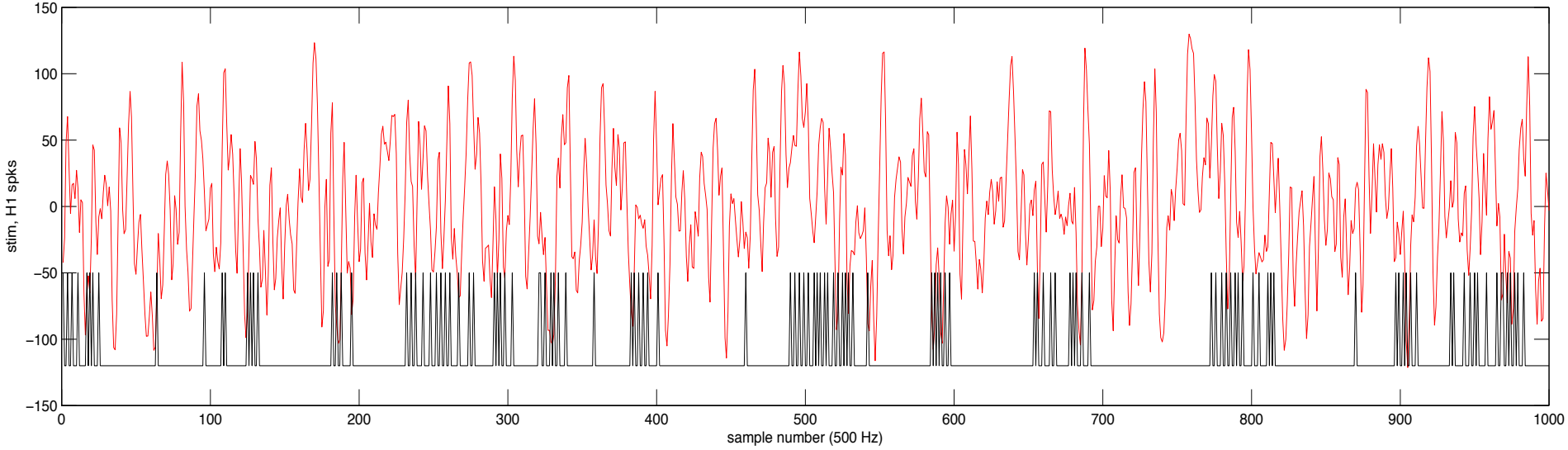


Single H1 neuron in lobula plate of each hemisphere

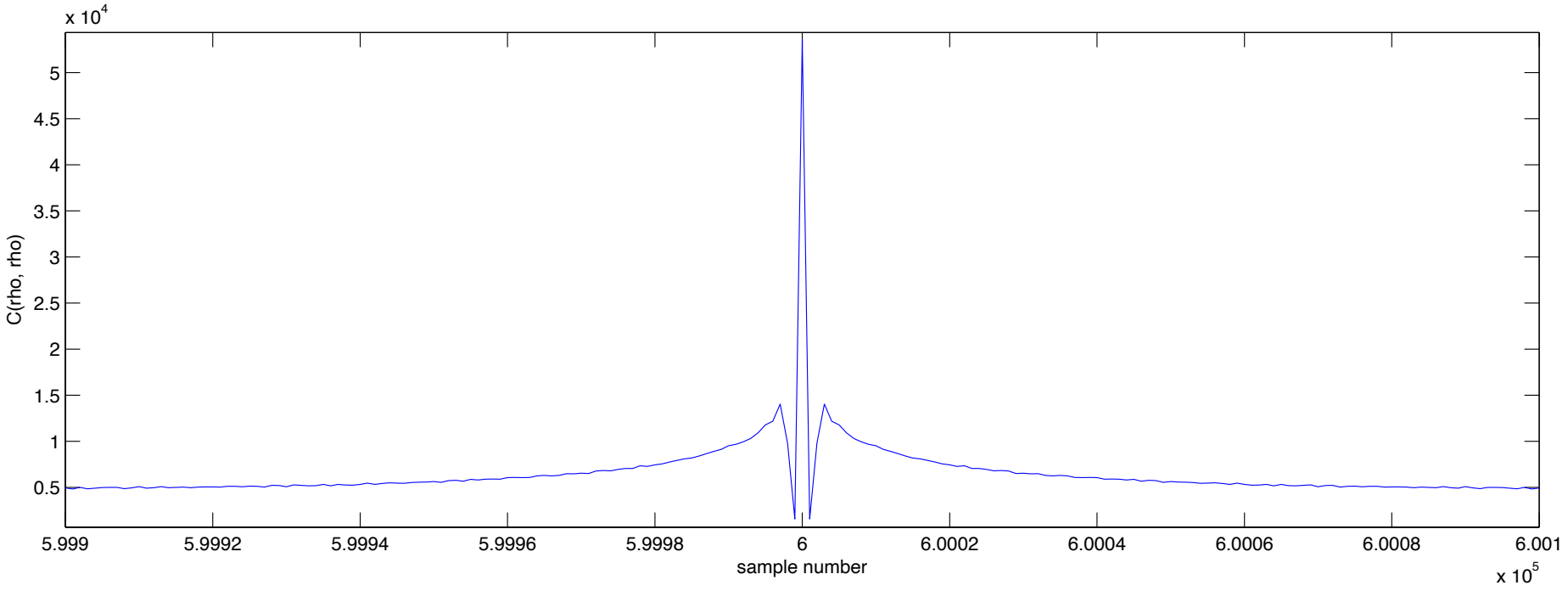
Lobula plate: highest visual area, before motor output.

H1 response during horizontal visual motion

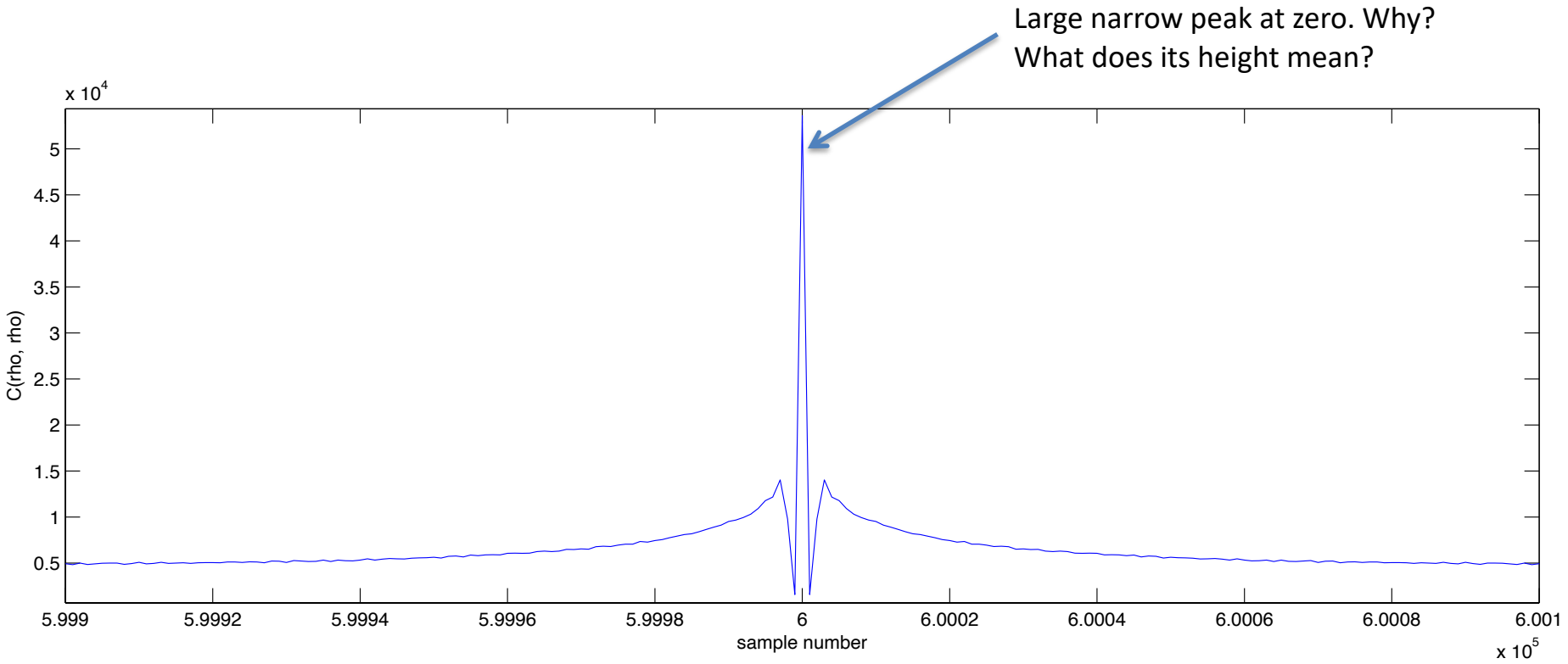
- Data: Rob de Ruyter van Steveninck
- 500 Hz, spikes and whole-field horizontal motion stimulus



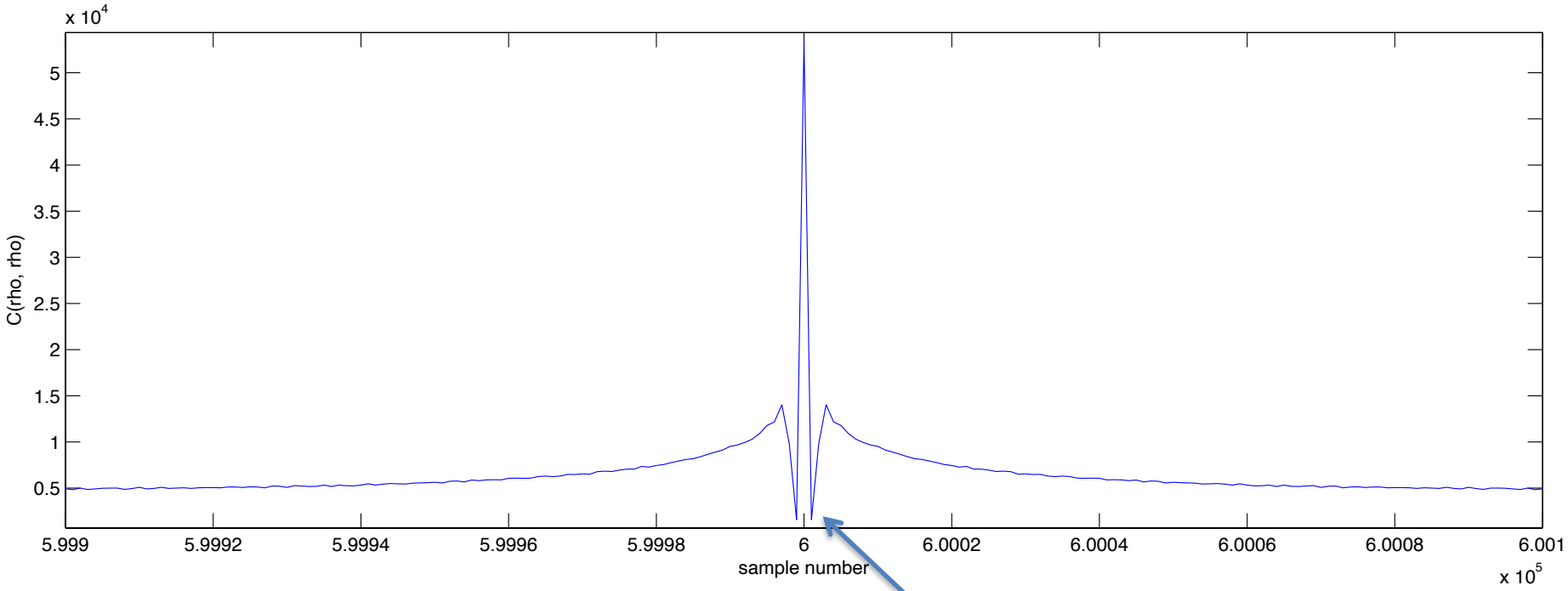
Spike autocorrelation



Spike autocorrelation

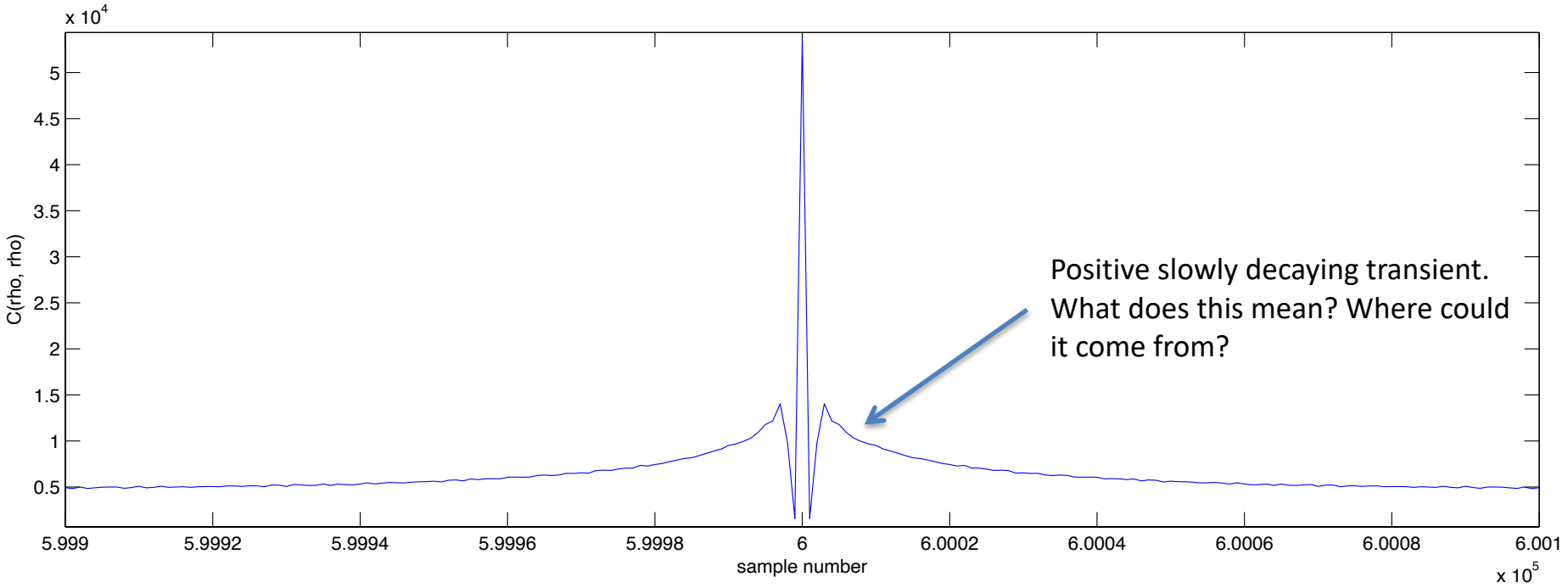


Spike autocorrelation

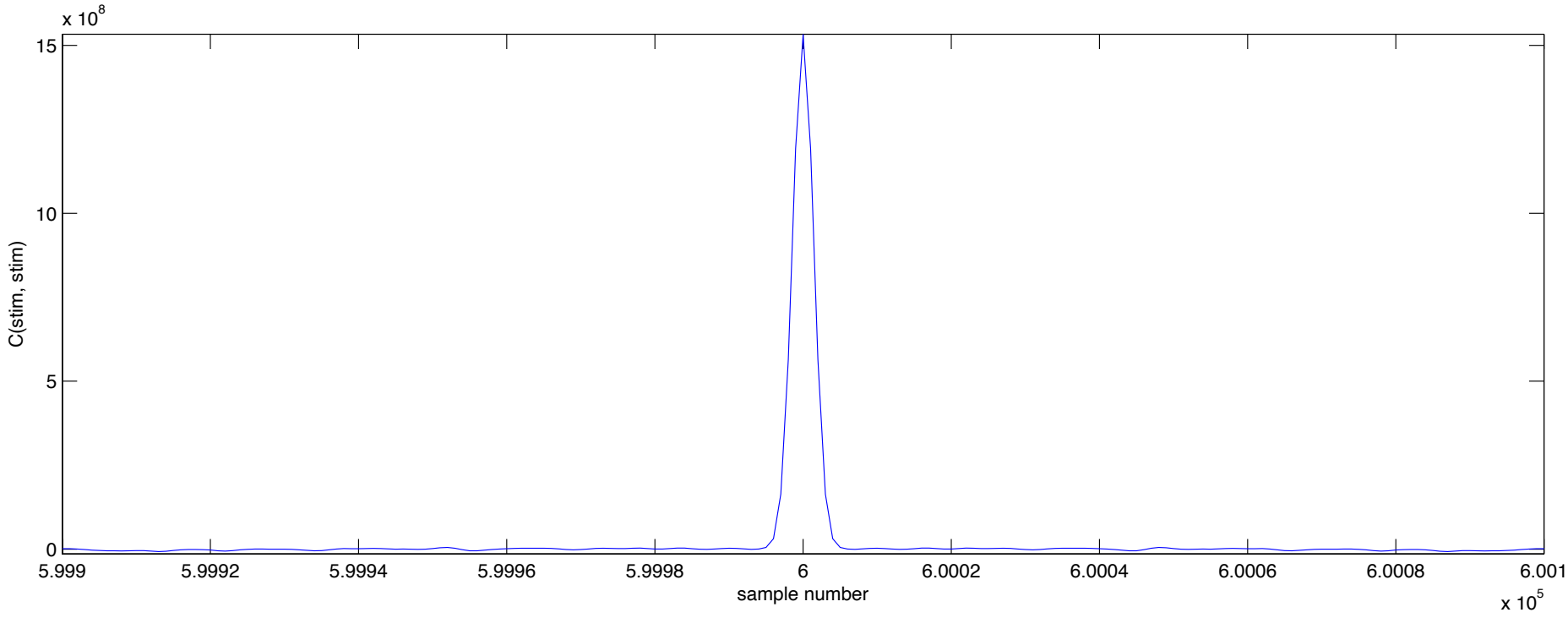


Sharp narrow dip. What does this mean?
Where could it come from?

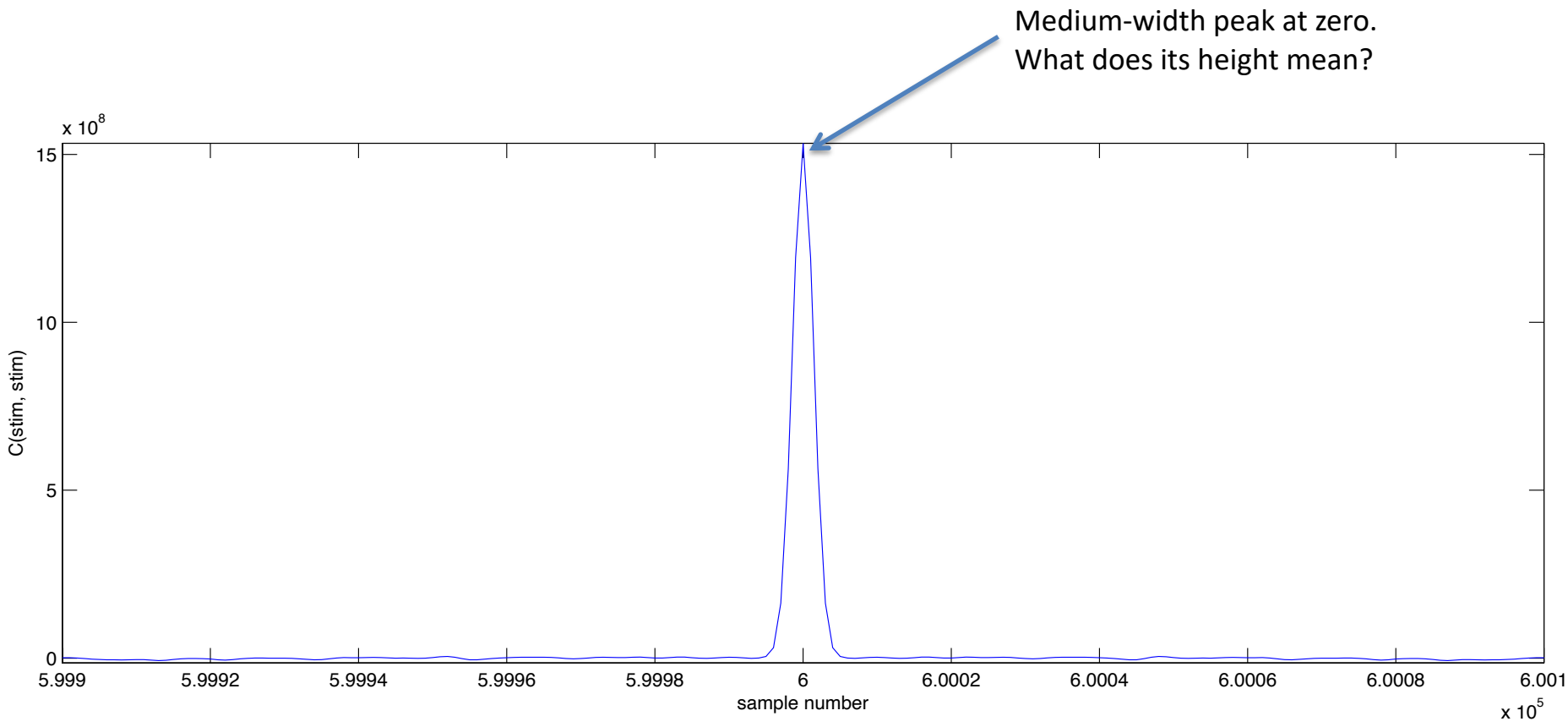
Spike autocorrelation



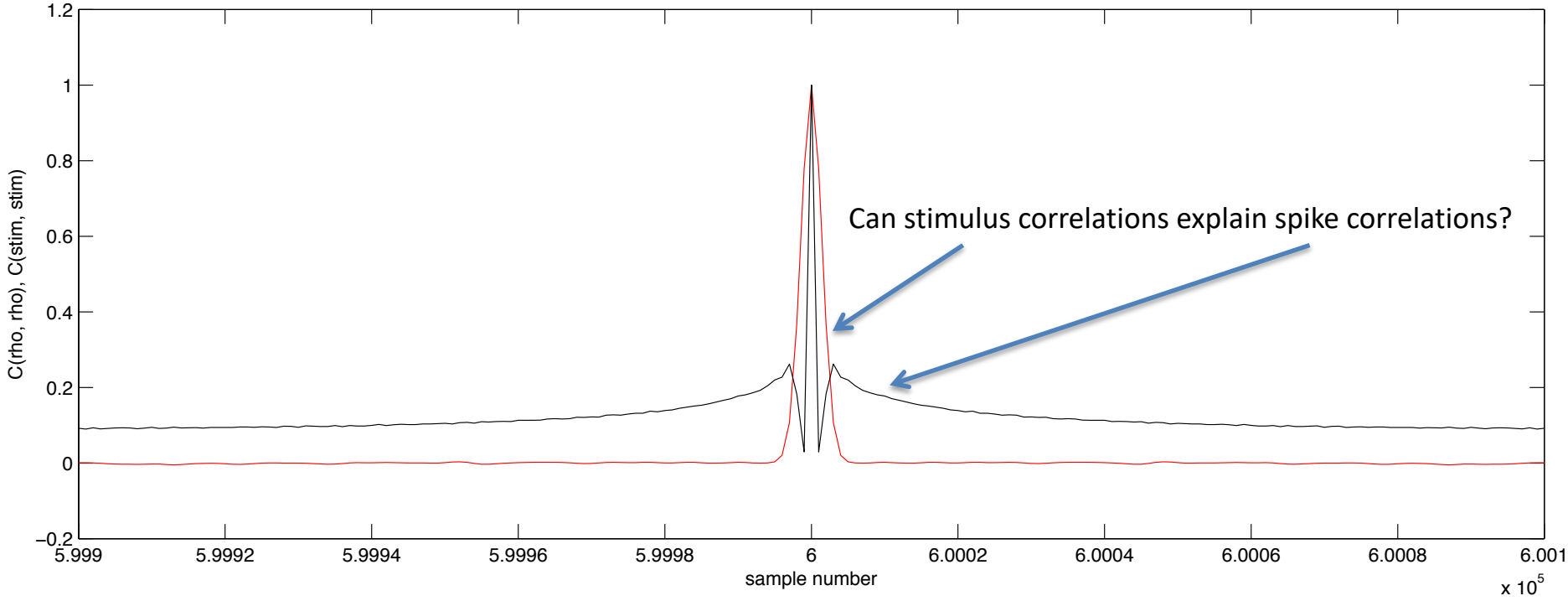
Stimulus autocorrelation



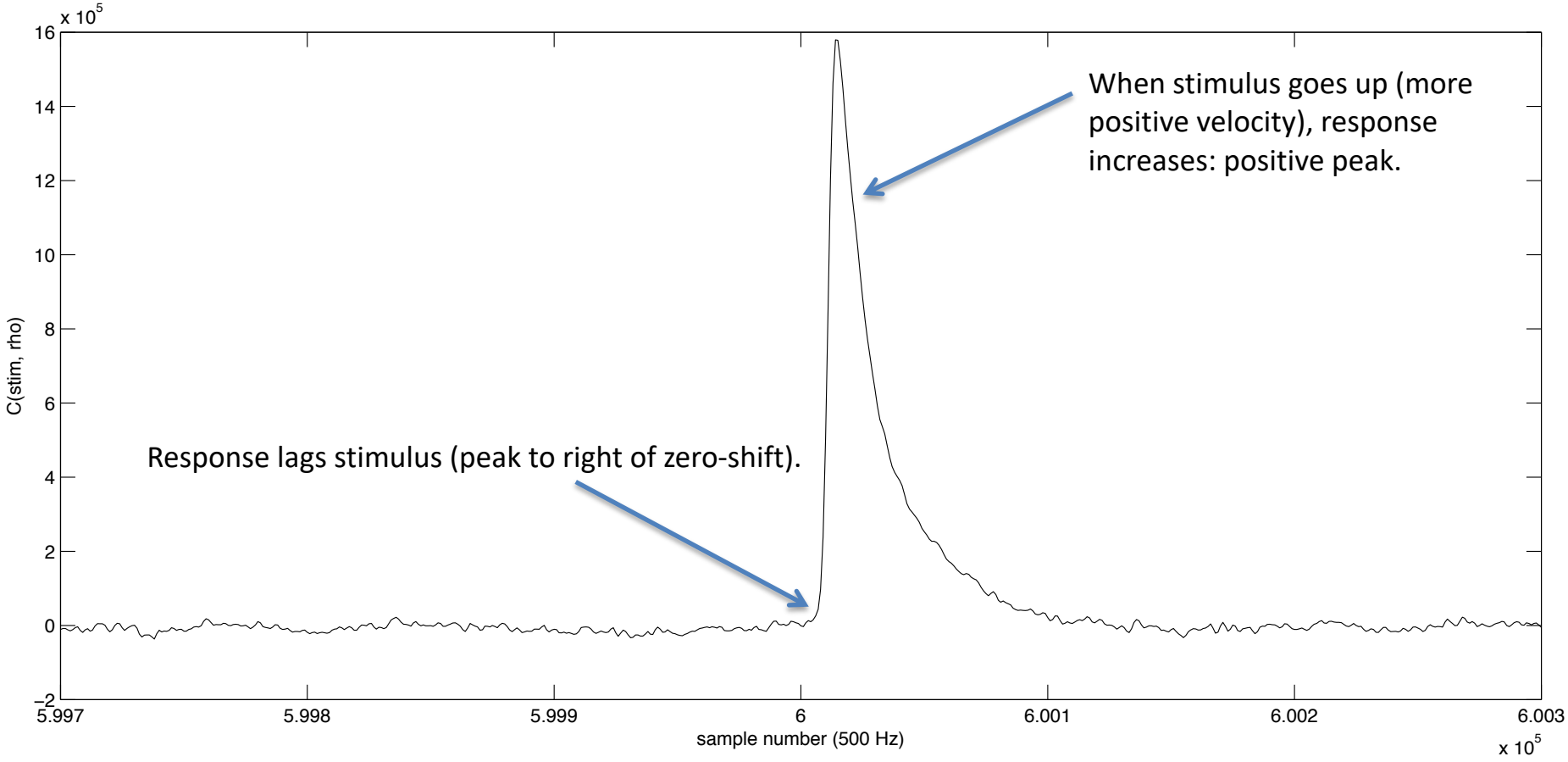
Stimulus autocorrelation



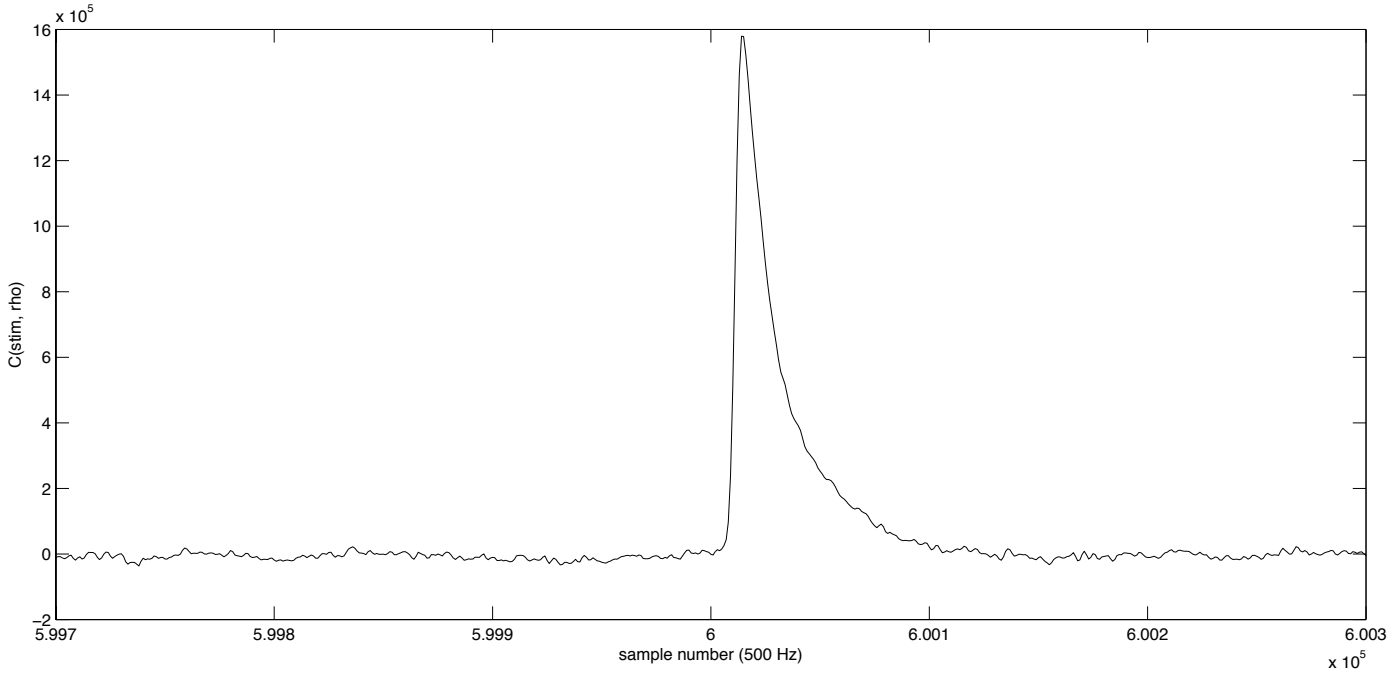
Stimulus, spike autocorrelations



Stimulus, spike cross-correlation



Stimulus, spike cross-correlation



What does stimulus, response cross-correlation really mean, from modeling perspective?

Back to original goal: Modeling

WHAT DOES IT MEAN TO BUILD A MODEL OF OBSERVATIONS IN THIS EXPERIMENT?

Modeling

- Relatively simple/compact description of data, good prediction performance.
- Extracting “features” of data as a way to model it.
- To determine predictability, important to cross-validate models/fits.

Modeling spike train data

Model: Simple, predictive description. But of what?

- Given stimulus, predict spikes?

- Given spikes, “predict” stimulus?

Modeling spike train data

Model: Simple, predictive description. But of what?

- Given stimulus, predict spikes? *Encoding* model
- Given spikes, "predict" stimulus? *Decoding* model

Yes, both!

Summary

- Autocorrelations of stimulus, response tell us about structure within stimulus, response.
- Comparison of the autocorrelations helps understand differences in the structure and motivates us to search for causes for these differences.
- Cross-correlation tells us about some relationships between stimulus and response: time-lags, sign of relationship, etc.
- Better understanding of what stimulus, response cross-correlation is telling us?