Syllabus for M394C - Mathematical neuroscience

Topics in neural dynamics, information theory, and machine learning

1 Course information

Instructor: Thibaud Taillefumier, ttaillef@austin.utexas.edu
Lectures: Tuesday and Thursday, 9:30AM-11:00AM, RLM 10.176.
Office hours: Monday 2:00PM-3:00PM, Wednesday 12:00PM-1:00PM:, RLM 10.148
Webpage: http://ctcn.utexas.edu/mathematical-neuroscience

2 Course objective

This course is intended for mathematicians interested in neuroscience and mathematically-inclined computational neuroscientists. The emphasis will be primarily on the analytical treatment of neuroscience-inspired models and algorithms. The objective of the course is to equip students with a solid technical and conceptual background to tackle research questions in mathematical neuroscience.

The course will be structured in three blocks: neural dynamics, information theory, and machine learning.

2.1 Neural dynamics

Neural computations emerge from myriads of neuronal interactions occurring in intricate networks that have evolved over eons of time. Due to the obscuring complexity of these networks, we can only hope to uncover principles for neural computations through the lens of mathematical modeling and analysis. The main theoretical challenge is to relate quantitatively structure and activity in a tractable way, i.e. to uncover hierarchies of low-dimensional representations for the activity of high-dimensional neural systems. In this block, we will present attempts made in that direction while introducing the mathematical formalisms associated to classical models of neural dynamics. Specifically: $i)$ We will characterize distinct dynamical regimen of neural activity in deterministic single-cell models and in deterministic population models. $ii)$ We will analyze neural variability in stochastic neural networks modeled via point-processes (i.e. intensity-based models) or via diffusion processes (i.e. integrate-and-fire models). $iii)$ We will examine network dynamics in various simplifying mean-field limits, including the traditional thermodynamics mean-field limits but also the replica mean-field limit. To complete this program, we will mostly rely on tools from the theory of dynamical systems and stochastic calculus (bifurcation theory, Markovian and stationary analysis).

References

[1] Eugene M. Izhikevich. *Bifurcations in brain dynamics*. ProQuest LLC, Ann Arbor, MI, 1996. Thesis (Ph.D.)–Michigan State University.

[2] Paul C. Bressloff. *Waves in neural media*. Lecture Notes on Mathematical Modelling in the Life Sciences. Springer, New York, 2014. From single neurons to neural fields.

[3] D. J. Daley and D. Vere-Jones. *An introduction to the theory of point processes. Vol. I*. Probability and its Applications (New York). Springer-Verlag, New York, second edition, 2003. Elementary theory and methods.

[4] D. J. Daley and D. Vere-Jones. *An introduction to the theory of point processes. Vol. II*. Probability and its Applications (New York). Springer, New York, second edition, 2008. General theory and structure.

[5] Ioannis Karatzas and Steven E. Shreve. *Brownian motion and stochastic calculus*, volume 113 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1991.

2.2 Information theory

To elucidate brain structure conceptually, it is tempting to look for "design principles" that would guide the development and the evolution of neural systems. Such a putative design principle is offered by the "efficient coding hypothesis", which states that sensory systems have evolved to optimally transmit information about the natural world given limitations on their biophysical components and constraints on energy use. In this block, we will introduce the theoretical framework suitable for investigating the efficient coding hypothesis from a mathematical standpoint. $i)$ We will start by reviewing the foundations of Shannon's information theory and its modern application to information processing in neural-network models: $a)$ We will present classical information-theoretic optimization results, e.g. rate-distortion theory, and some of their more recent variants, e.g. information bottleneck, as well as the corresponding optimization algorithms. $b)$ We will introduce maximum-entropy methods for statistical inference about neural networks in the framework of information geometry. $ii)$ Then, we will explore some outstanding information-theoretical problems in neuroscience (channel optimization, entropy production). This block will rely on results from constrained optimization theory (essentially the KKT conditions) and will require some elementary notions of variational and differential calculus.

References

[1] Thomas M. Cover and Joy A. Thomas. *Elements of information theory*. Wiley-Interscience [John Wiley & Sons], Hoboken, NJ, second edition, 2006.

[2] Fred Rieke, David Warland, Rob de Ruyter van Steveninck, and William Bialek. *Spikes*. A Bradford Book. MIT Press, Cambridge, MA, 1999. Exploring the neural code, Computational Neuroscience.

[3] E. T. Jaynes. *Probability theory*. Cambridge University Press, Cambridge, 2003. The logic of science, Edited and with a foreword by G. Larry Bretthorst.

[4] Shun-ichi Amari and Hiroshi Nagaoka. *Methods of information geometry*, volume 191 of *Translations of Mathematical Monographs*. American Mathematical Society, Providence, RI; Oxford University Press, Oxford, 2000. Translated from the 1993 Japanese original by Daishi Harada.

2.3 Machine learning

Machine learning has allowed the realization of speech recognition, language translation, natural-object recognition, and self-driving cars. These achievements, which rival human performance, are performed by neural networks that mimic many structural features of the brain and learn how to perform tasks via biologically inspired rules, such as reinforcement learning. However, the mathematical theory underlying this computational feats is still in its infancy. This block will present the mathematical theory supporting a few machine learning methods in supervised learning, in reinforcement learning, and in unsupervised learning. Specifically: i) We will present the theory of reproducing-Hilbert-kernel spaces (RHKSs) underlying support vector machines (SVMs). ii) We will introduce the theory of Markov decision processes (MDPs) in the context of reinforcement learning (successor representation). ii) We will discuss the probabilistic framework of recent generative models in artificial intelligence, namely the autoencoder networks and the generative-adversarial networks (GANs).

References

[1] Bernhard Scholkopf and Alexander J. Smola. *Learning with Kernels: Support Vector Machines, Regularization, Optimization, and Beyond.* MIT Press, Cambridge, MA, USA, 2001.

[2] Richard S. Sutton and Andrew G. Barto. *Introduction to Reinforcement Learning*. MIT Press, Cambridge, MA, USA, 1st edition, 1998.

3 Tentative schedule

January 22: Introduction

January 24: Hodgkin-Huxley/Reduced models.

January 29: Local bifurcation analysis 1.

January 31: Local bifurcation analysis 2.

February 5: No class.

February 7: Bifurcations of two-dimensional neurons.

February 12: Intensity-based neural models.

February 14: Integrate-and-fire neural models.

February 19: Thermodynamic mean-field limits.

February 21: Replica mean-field limits.

February 26: Maximum entropy methods.

February 28: Information geometry 1.

March 5: Information geometry 2.

March 7: Mutual information.

March 12: Rate-distortion theory.

March 14: **Midterms**

March 26: Information bottleneck.

March 28: Variational Fisher information.

April 2: Linear separability/Perceptron algorithm.

April 4: Linear separability and combinatorics.

April 9: Reproducing-kernel-Hilbert space.

April 11: Support vector machine.

April 16: Markov-decision process.

April 18: Dynamic programming.

April 23: Q-learning algorithm.

April 25: Reduction to linear Bellman equations.

April 30: Autoencoder networks.

May 2: Generative adversarial networks.

May 7: **Projects**

May 9: **Projects**

4 Grading of the course

There are three components to your grade: problem sets, a midterm, and a final project. The breakdown will put equal weights on each components ($1/3$, $1/3$, $1/3$). Problem sets will be posted on the course website every other Thursdays and will be due two weeks after being posted. The midterm exam will be given in class on March 14. The projects will be in-class presentations of mathematical proofs or research papers, which will be assigned in the second half of the semester.

5 Academic honesty

Students are expected to behave with integrity. The sanction for any student found in violation of the UT Honor Code is to be decided by the instructor. For more information please go the following website: deanofstudents.utexas.edu/sjs/acint_student.php.

6 Religious holidays

A student who misses classes or other required activities, including examinations, for the observance of a religious holiday should inform the instructor as far in advance of the absence as possible, and no less than 2 weeks in advance, so that arrangements can be made to complete an assignment within a reasonable time after the absence.

7 Students with Disabilities

The University of Texas provides upon request appropriate academic accomodations for qualified students with disabilities. For more information contact Services for Students with Disabilities: ddce.utexas.edu/disability, 471-6259, 471-6441 TTY.